of the order conditions, which is obtained by a simple transcription of the structure of the corresponding graph.

The stable computation of trajectories of N-body problems is important for astronomical applications as well as for studies of atomic systems. The article "Reversible adaptive regularization: perturbed Kepler motion and classical atomic trajectories" by B. Leimkuhler discusses the impact of symplectic and time-reversible integrators on the energy error, it studies time transformations and the use of variable step sizes, and it presents a series of interesting numerical experiments with a new code that is especially written for the simulation of perturbed Kepler motions and classical atomic trajectories.

To sum up, this issue shows several important aspects of the wide field of geometric integration, all of which are written by experts in this topic.

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7[**37Mxx, 65Pxx**]—*Dynamical systems and numerical analysis*, by A. M. Stuart and A. R. Humphries, Cambridge University Press, New York, NY, 1998, xxi+685 pp., 23 cm, hardcover, \$64.95, softcover \$39.95

To come to the point first: This is an extraordinarily well-made monograph on dynamical systems which are generated by application of linear multistep methods and Runge–Kutta methods to explicit ordinary differential equations (ODEs). In particular, emphasis is put on the qualitatively correct reflection of properties, resp. the structure of the dynamical system generated by the ODE itself.

The principle aim of this book is—according to the two authors—to address two questions:

I. Assume that the differential equation has a particular invariant set. Does the numerical method have a corresponding invariant set which converges to the true invariant set as $\Delta t \to 0$? If so, what is the rate of convergence?
II. Assume that the vector field defining the differential equation has a particular structural property which confers certain properties on the dynamical behaviour of the equation. Find special numerical methods which inherit these structural properties under mild or no restrictions on the time-step or, for general numerical methods, find conditions on Δt under which these structural properties are inherited.

During the last 15 years, considerable progress has been made in answering these questions. Numerous articles have been written, by numerical mathematicians as well as analysts, among them, not least the two authors of the present monograph. In this 700-page monograph they collect these results to form a comprehensive and cogent account which "is intended to be accessible to anyone familiar with either dynamical systems or numerical analysis theory and, with a little work, to someone familiar with neither."

No doubt, this book is a considerable gain for all those who want to get familiar with this field of mathematics as well as for someone requiring a reference text for the major results in the field. However, someone looking for a guide for the immediately practicable numerical solution of initial value problems or for the appropriate modelling of genuine initial value problems should better consult other sources.

The ODEs allowed here are always explicit and autonomous where the concerned vector field is globally defined on $\mathbb{R}^p$ and Lipschitz-continuous, resp. locally Lipschitz-continuous with additional global structural assumptions. This is the price that has to be paid for the really nice theory, which would not have been possible otherwise.

As far as numerical integration methods are concerned, exclusively constant step-sizes are assumed, if necessary sufficiently small ones with a relatively detailed specification of this condition. In case of implicit integration methods it is assumed that the resulting nonlinear equations can be solved exactly. Problems occurring in practice, like rounding errors, defects and error estimations, are not an issue here.

I read the book with mixed feelings. On the one hand, I realized the very large distance from the starting point and from the intention of the investigations the authors explain in the preface: "Dynamical systems are pervasive in the modelling of naturally occurring phenomena. Problems as diverse as the simulation of planetary interactions, fluid flow, chemical reactions, biological pattern formation and economic markets can all be modelled as dynamical systems. The theory of dynamical systems is concerned primarily with making qualitative predictions about the behaviour of systems which evolve in time, as parameters which control the system, and the initial state of the system itself, are varied. Most of the models arising in practice cannot be completely solved by analytic techniques and thus numerical simulations are of fundamental importance in gleaning understanding of dynamical systems. Hence it is crucial to understand the behaviour of numerical simulations of dynamical systems in order that we may interpret the data obtained from such simulations and in order to facilitate the design of algorithms which provide correct qualitative information without being unduly expensive. These issues are at the heart of this book." Problems of in how far or where the mathematically useful structural properties are relevant for the real world are not touched. Neither is it mentioned which out of these lots of nice results are applicable and useful in practice. On the other hand, I take pleasure in the beauty and high degree of perfection the mathematical theory of dynamical systems generated by numerical integration formulas as documented in this monograph has obviously now reached.

The monograph is properly divided into the following chapters:

1. Finite dimensional maps
2. Ordinary differential equations
3. Numerical methods for initial value problems
4. Numerical methods as dynamical systems
5. Global stability
6. Convergence of invariant sets
7. Global properties and attractors under discretization
8. Hamiltonian and conservative systems

All chapters are completed by a detailed and competent discussion of references. Various simple and well-obvious examples serve for illustration and motivation.

Based on their experience, the authors provide hints for lectures as well as numerous exercises, which is valuable for teachers as well as students.

The book is admirably carefully written. The unreasonable references (e.g., "Proof of Theorem XY: See Exercise MN/Exercise MN: Proof of Theorem XY"), however, I found somewhat bothersome. Otherwise, the representation is clear and as didactically well structured as can already be assumed from the subdivision into the eight chapters mentioned. A pleasant layout contributes to the pleasure one surely has reading the book.

Roswitha März

8[65-02, 65Fxx]—*Computer solution of large linear systems*, by Gerard Meurant, North Holland, Amsterdam, 1999, xxii+753 pp., 23 cm, hardcover, $149.50

Two characteristics of this book are immediately evident: the table of contents is very detailed and the list of references is large, which makes the book a useful reference for someone working in the area. The chapters are well organized and it is easy to find something quickly through the table of contents if you know the name of a technique, or through browsing in the appropriate chapter if not. The index in the back of the book is rather sparse and not very helpful.

This book contains many results on estimates of eigenvalues and error bounds for symmetric positive definite systems. Some proofs are presented in each chapter to give an idea of the typical arguments used. For many theorems the reader is referred to the original work for the proof. This allows the author to present a lot of material without making the book overly large. This is not a defect as most researchers are interested in the original citation anyway. Also, since the book carries a fairly complete and up-to-date (for the mid- to late-1990s) set of algorithms with descriptions of implementations, it is easy to search in the appropriate chapter and answer the question, "Who has done work in this area before?"

This book is not intended as a text as there are no exercises, and if a proof is not presented, it tells you where to find it. Some of the material is developed in detail and is easy to follow for someone new to the field. Some of the results are not as well motivated.

Chapter 1 covers introductory material, beginning with topics in linear algebra and graph theory and progressing to issues involved in solving linear systems on computers, where exact arithmetic is not available. Sparse storage schemes, a short history on BLAS routines, and parallel machines are also presented.

The second chapter is devoted to Gaussian elimination on dense matrices. Six different algorithms are presented, which may be a bit of overkill. A lot of attention is paid to tridiagonal and block tridiagonal systems, which are the types of large systems one would solve this way. Otherwise, the chapter is rather brief. Some parallel implementation details are presented at the end.

Chapter 3 covers Gaussian elimination for sparse matrices. Graph theory is used to motivate reordering strategies. Most of this chapter is devoted to symmetric positive definite matrices, with just a little bit on nonsymmetric matrices. Admittedly, this is where the theory is best developed. There is some discussion of parallel implementations also.

Chapter 4 is a rather short chapter on fast solvers for separable PDEs. It covers mostly FFT and cyclic reduction methods.

Chapter 5 contains the classical iterative methods, such as Jacobi, Gauss–Seidel, SOR, and variations. Convergence criteria are derived, as well as convergence rates